

# Reply to “Comment on ‘Models of Stochastic, Spatially Varying Stress in the Crust Compatible with Focal-Mechanism Data, and How Stress Inversions Can Be Biased toward the Stress Rate’ by Deborah Elaine Smith and Thomas H. Heaton” by Jeanne L. Hardebeck

by Deborah Elaine Smith\* and Thomas H. Heaton

## Introduction

In her comment (Hardebeck, 2015) on our stress heterogeneity article (Smith and Heaton, 2011), Hardebeck suggests a different focal-mechanism error distribution than what we used in our 2011 article and suggests that this new error distribution will reduce our estimates of stress heterogeneity. In response to this, we have rerun our calculations three ways: (1) with the original mechanism error distribution from Smith and Heaton (2011), (2) with a mechanism error distribution similar to the one presented by Hardebeck (2015), and (3) with a mechanism error distribution derived from repeating earthquake statistics. We find the two new mechanism error models, relative to the original mechanism error distribution, reduce the heterogeneity ratio ( $HR$ ) estimates by approximately 35%–40% (using Hardebeck’s suggested distribution) and by approximately 8%–10% (using the repeating earthquake based error distribution).

Applying these two new mechanism error distribution models helps parameterize the estimates of stress heterogeneity amplitude but does not change the main novel points of the Smith and Heaton (2011) article. Namely, we find that focal-mechanism data are still compatible with a heterogeneous stress that is more dissimilar at large interevent distances and more correlated at small interevent distances and that a heterogeneous stress can bias traditional stress inversions toward the stressing rate function.

Last, we demonstrate that as the size of the stress inversion region decreases and as the maximum variability of the heterogeneous stress decreases, the normalized stress inversion bias also decreases. This is consistent with taking the model of Smith and Heaton (2011) to the limit where region size decreases to a point source; however, most stress inversions may require dimensions closer to the outer scale of the stress (~60 km for southern California) and hence experience significant stress inversion biasing toward the stressing rate.

Hardebeck (2015) also refers to her 2010 article (Hardebeck, 2010) as a basis for refuting biasing of stress inversions. We disagree with Hardebeck’s stated refutation of stress inversion orientations in Smith and Heaton (2011) and

Smith and Dieterich (2010); however, addressing the Hardebeck (2010) article is best accomplished via a direct modeling test in a separate comment that takes into account the complexities of the nucleation process, including rapidly evolving slip over the period of days.

## Background

Smith and Heaton (2011) introduced a 3D stochastic model of stress heterogeneity with power-law-like scaling for heterogeneity amplitude. When coupled with a stressing rate and a plastic yield criterion, this model was used to generate synthetic earthquake focal mechanisms to simulate background seismicity. To compare the synthetic focal mechanisms with real data, they added focal-mechanism error with a normally distributed rotation amplitude centered on zero and uniform randomly distributed rotation axes. The statistics of these synthetic focal mechanisms with modeled measurement error were compared with the southern California data set by Hardebeck and Shearer (2003) to invert for estimated stress heterogeneity parameters in southern California. The parameter of particular interest is  $HR$ , as defined in equation (2) of Smith and Heaton (2011), in which  $HR$  measures the amplitude of the stress variation relative to the mean stress level.

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The key conclusions in Smith and Heaton (2011) are as follows. First, similar to one of the Hardebeck (2006) observations, Smith and Heaton (2011) found the average angular difference ( $AAD$ ) decreased with decreasing interevent distance when comparing pairs of synthetic focal mechanisms generated by their stochastic stress model. This implies the power-law filtering generates stress that is fairly uncorrelated at large distances and is increasingly correlated or similar at smaller distances.

Second, they found that when one applies a failure criterion to generate events from a stress grid with spatially variable initial stress and a spatially uniform loading rate, the events tend to cluster toward the loading rate direction. In other words, by treating earthquakes as a critical phenomenon, the points that preferentially fail already have a prestress

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orientation somewhat aligned with the loading stress. This produced a biased sampling, in contrast to a simple uniform random sampling of the stress in the grid.

This second finding has two implications. The events tend to cluster more tightly than one would calculate for a uniform random sampling of the stress grid. This can lead to underestimates of the stress heterogeneity parameter,  $HR$ , if the tight clustering is not properly taken into account. If a spatially uniform loading stress and the spatial mean of prestress are misaligned, there can be a bias in the focal-mechanism event orientations that are more consistent with the loading stress orientation.

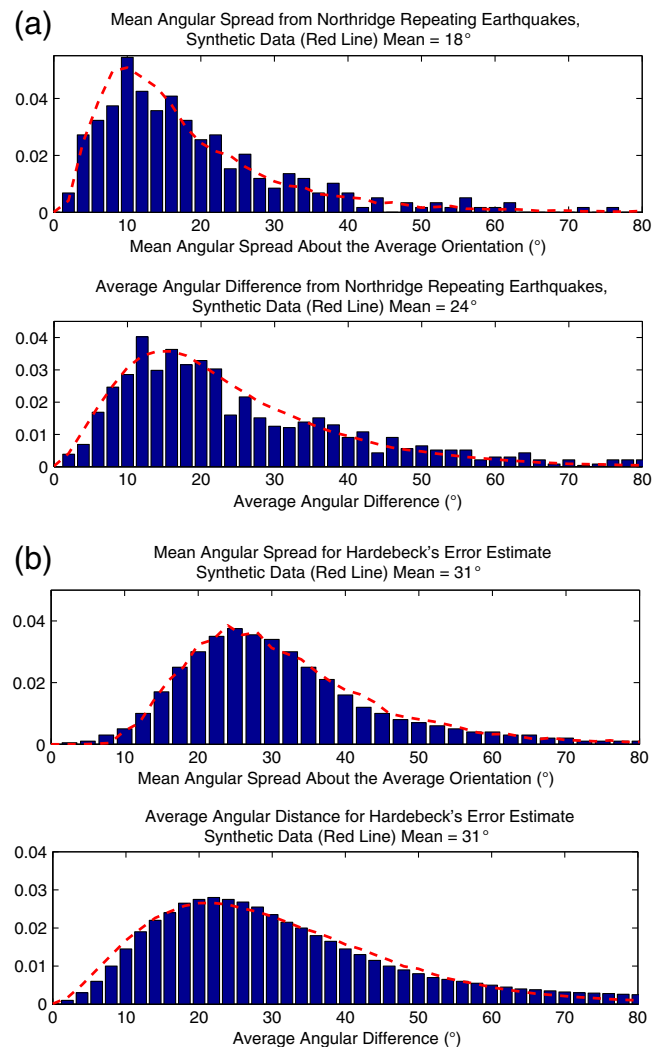
### Error Distributions for Modeling Focal-Mechanism Uncertainty

As pointed out by Hardebeck (2015), taking into account the geometry of the problem generates an estimated error distribution for modeled focal-mechanism uncertainty that has a peak offset from zero, in contrast with our normal distribution with a peak centered at zero. The only questions are: What does that distribution look like and to what degree is it offset from zero? Then to what degree does this new distribution affect our estimates of stress heterogeneity,  $HR$ ?

To answer these questions, we use three models of focal mechanism error to generate noisy synthetic focal mechanisms. These three error models include: (1) our original normal error distribution, (2) a distribution similar to what Hardebeck (2015) presents, and (3) a distribution based on Shearer and Hardebeck's Northridge repeating earthquake aftershock data (Shearer *et al.*, 2003). Then by comparing noisy synthetic mechanisms with mechanisms from Shearer and Hardebeck (see Data and Resources) we recalculate our estimates of stress heterogeneity ( $HR$ ) and percent bias in light of these three mechanism error models.

The first new error distribution we explore, based on the Hardebeck (2015) suggestion, may be representative of the uncertainty for individual focal-mechanism measurements; therefore, we choose it as the upper bound for our uncertainty estimates. However, the calculations that estimated  $HR$  in figure 9 of Smith and Heaton (2011), based on data from Hardebeck (2006), determined the best fit to AADs between pairs of focal mechanisms. So the question arises: To what extent could the relative focal-mechanism error between pairs of mechanisms be significantly less than this upper bound, especially for closely spaced mechanisms?

To address this question, we choose the second new error distribution, based on statistics of repeating earthquake clusters, as a means of estimating relative focal-mechanism error. The idea is fairly simple. Because repeating earthquakes have similar waveforms and occur essentially at the same location, any variation in the mechanisms for a particular cluster can be approximated to first order as relative measurement uncertainty. To test this, Shearer and Hardebeck were willing to share with us their Northridge aftershock repeating cluster catalog from Shearer *et al.* (2003). From this



**Figure 1.** (a) Mean angular spread mechanism distribution and average angular difference (AAD) from repeating earthquake clusters in the Northridge aftershock sequence (Shearer *et al.*, 2003). Only A and B quality events were selected, with a minimum cluster size of three events each. Modeled mechanism error statistics are plotted with dashed lines using the parameterization explained in the text. (b) Similar histograms are plotted from the Hardebeck (2015) data. Modeled mechanism error statistics are again plotted with dashed lines using a parameterization explained in the text. The color version of this figure is available in the electronic edition.

catalog, we extract A and B quality events (the same quality level used in our 2011 study and in Hardebeck's study), choose clusters with a minimum of three events, and calculate the mean angular spread for each cluster. From a total of 294 events, we have the following distribution of mechanism error as shown in Figure 1a. It has a mean angular spread of approximately 19°, which is 5° more than the normal distribution studied by Smith and Heaton, and an AAD of approximately 27°, which is very similar to the minimum AAD seen by Hardebeck (2006) at distances less than 100 m.

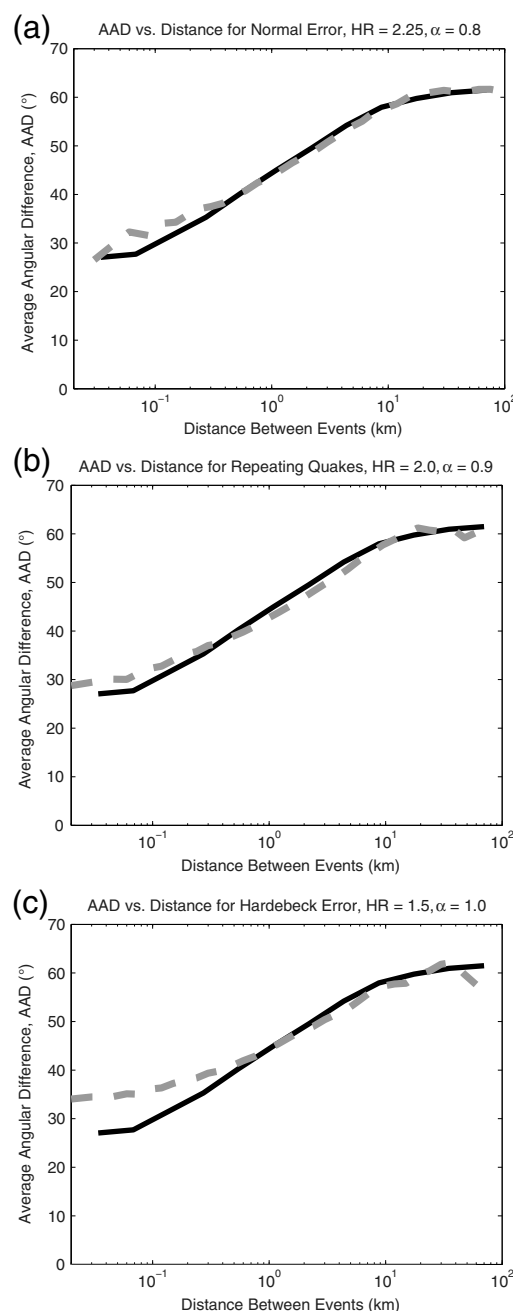
In modeling the focal-mechanism error, we represent each rotation error as a rotation amplitude  $\omega$  about a rotation axis  $(\theta, \phi)$ . For the normal distribution error in Smith (2006)

and Smith and Heaton (2011), we had a normally distributed  $\omega$  and uniform random  $(\theta, \phi)$ . For the repeating earthquake distribution statistics in Figure 1a and for the Hardebeck (2015) estimate of focal-mechanism error in Figure 1b, we apply lognormal statistics for the rotation amplitude  $\omega$  and Von-Mises distributions for the rotation axes  $(\theta, \phi)$ . This has the effect of offsetting the rotation amplitude from zero, using the lognormal statistics and clustering the error rotations due to the circular statistics on the rotation axes. In Figure 1a, when simulating repeating earthquake statistics, we applied a  $\mu = 2.7$  and  $\sigma = 0.67$  for the  $\omega$  lognormal statistics and a  $\theta = 0$  and  $\kappa = 1.0$  for each angle in the rotation axis  $(\theta, \phi)$ . In Figure 2a, when simulating the Hardebeck (2015) error statistics, we applied a  $\mu = 3.375$  and  $\sigma = 0.41$  for the  $\omega$  lognormal statistics and a  $\theta = 0$  and  $\kappa = 2.25$  for each angle in the rotation axis  $(\theta, \phi)$ . Note that the lognormal random numbers were drawn via the standard MATLAB toolbox, and the circular random numbers were generated by Beren's *circ\_ymrnd* MATLAB toolbox (2006; see Data and Resources).

### How the Error Distributions Affect Stress Heterogeneity Amplitude, $HR$ , $\alpha$ , and Percent Biasing toward the Stressing Rate Orientation

In Figure 2, we examine the synthetic AAD as a function of focal-mechanism pair separation distance to invert for  $HR$  and  $\alpha$  as in figure 9 of Smith and Heaton (2011). This plot is based on AAD statistics from Hardebeck (2006) for A and B quality southern California focal-mechanism data. We do this for each error distribution by showing the best-fitting synthetic solutions, using a grid-search technique similar to but simpler than Smith and Heaton (2011). This grid search focuses on the parameter space close to the solution and ignores location error for 2D grids. The best-fitting synthetic with normal distribution error is shown in Figure 2a, with  $\alpha = 0.8$  and  $HR = 2.25$ ; the best-fitting synthetic with repeating earthquake error is shown in Figure 2b, with  $\alpha = 0.9$  and  $HR = 2.0$ ; and the best-fitting synthetic with Hardebeck's error is shown in Figure 2c, with  $\alpha = 1.0$  and  $HR = 1.5$ .

These plots show three things. (1) The new error distributions generate smaller  $HR$ s. The best-fit synthetic with repeating earthquake error predicts an  $HR$  11% smaller than the estimate with normal error and 16% less than  $HR = 2.375$ , which was originally estimated by Smith and Heaton (2011). The best-fit synthetic with Hardebeck (2015) error predicts an  $HR$  33% smaller than the estimate with normal error and 37% smaller than the Smith and Heaton (2011) estimate. (2) The more error that is added to the focal-mechanism synthetics, the more spatial smoothing is needed to generate similar AAD curves, which results in higher  $\alpha$ . (3) It is impossible to generate the minimum of  $26^\circ$  AAD with Hardebeck's error as seen in Figure 2a, in which the minimum AAD for mechanism error alone is  $31^\circ$ – $32^\circ$ . Consequently, both distributions predict a small-to-moderate reduction in the estimate of  $HR$ , and the statistics of the repeating events



**Figure 2.** Approximations of the Hardebeck (2006) calculation of AAD between pairs of focal mechanisms as a function of inter-event distance for southern California A and B quality mechanisms (black lines) and the best-fitting synthetic focal-mechanism data from 2D grids given a particular mechanism uncertainty model (dashed gray lines). (a) Using a normal distribution centered on zero for mechanism uncertainty like Smith and Heaton (2011), we estimate the best-fitting parameters to be approximately  $\alpha = 0.8$  and  $HR = 2.25$ . (b) Using mechanism uncertainty similar to repeating earthquakes, we estimate the best-fitting parameters to be approximately  $\alpha = 0.9$  and  $HR = 2.0$ . (c) Using mechanism uncertainty similar to Hardebeck (2015), we estimate the best-fitting parameters to be approximately  $\alpha = 1.0$  and  $HR = 1.5$ ; however, because the minimum AAD for this distribution is approximately  $31^\circ$ , it is impossible to precisely fit the data at small interevent distances. (a)–(c) Figures were modified from Smith and Heaton (2011), which was originally modified from Hardebeck (2006).

may yield the most appropriate relative mechanism error distribution, especially for short interevent distances.

Smith and Heaton (2011) estimated the stress heterogeneity amplitude,  $HR$ , and percent biasing for 12 subregions in southern California, so we redo those estimates with our new focal-mechanism error distributions. One method that Smith and Heaton (2011) applied to estimate the  $HR$  value was the AAD between pairs of focal mechanisms. Generally, the noisier the data the larger the AAD, which encompasses both mechanism measurement error and underlying stress heterogeneity—hence, the need for good mechanism error estimates when using this statistic to estimate stress heterogeneity. In other words, the wrong model of error can cause either an under- or over-estimate of the stress heterogeneity.

Smith and Heaton (2011) developed a scaling plot that relates the AAD to the  $HR$ . To create this, suites of synthetic focal-mechanism catalogs with different values of  $HR$  and an assumed mechanism error distribution were employed. The scaling plot then allowed them to overlay the data for each region as horizontal lines in their figure 12a. This mapped out a range of  $HR$ , the values of which are compatible with southern California.

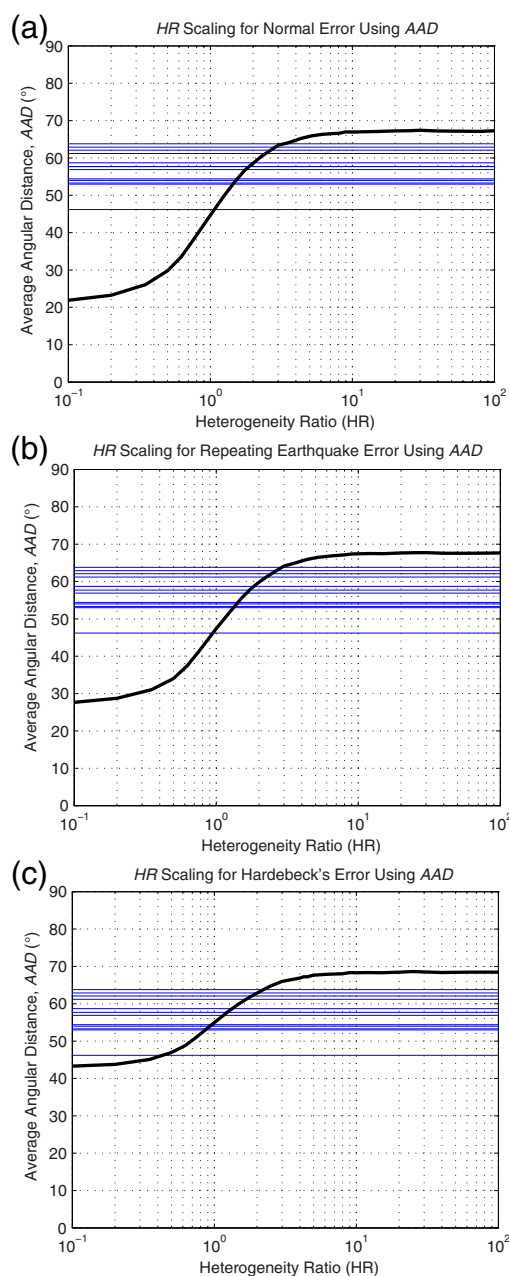
In this reply to Hardebeck's comment, we calculate  $HR$  scaling plots for three distributions. In Figure 3a, we show the scaling plot for our original normal distribution with zero mean; in Figure 3b, we show the scaling plot for the distribution suggested from the statistics of repeating earthquakes; and in Figure 3c, we show the scaling plot for a distribution similar to the one suggested by Hardebeck (2015). The intersection of the data (shown as horizontal lines) maps out the range of stress heterogeneity values observed for each assumed mechanism error distribution. The results are shown in Table 1.

We find that by updating the mechanism error model, the estimates of  $HR$  are reduced but by varying amounts. The mechanism error distribution suggested by repeating earthquake statistics generates  $HR$  values in the 0.96–2.62 range, approximately 8%–10% less than the original calculation. The mechanism error distribution suggested by Hardebeck (2015) generates  $HR$  values in the 0.44–2.20 range, approximately 35%–45% less than the original calculation.

When we compare these new  $HR$  estimates with the percent bias scaling plotted in figure 17 of Smith and Heaton (2011), we have the following estimates of percent bias in stress inversions. The distribution for mechanism error based on repeating earthquakes generates our upper bound for  $HR$  and biasing, where biasing is estimated in the 30%–60% range, with an average of about 51%. The distribution similar to the one suggested by Hardebeck (2015) generates our lower bound for  $HR$  and biasing, where biasing is estimated in the 15%–50% range, with an average of about 38%.

### The Scale Dependence of the Inversion Region for Stress Orientation Biasing

The other effect we investigate is how the power-law filtering of the stress affects stress inversion biasing. We note



**Figure 3.** How different mechanism error distributions affect the estimates of heterogeneity ratio ( $HR$ ). The solid black line shows the scaling relationship derived from calculating the AAD of synthetic focal mechanisms with varying  $HR$  and an assumed mechanism error distribution. The horizontal lines show the AAD of 12 regions in southern California, as listed in Table 1. The intersection of the horizontal lines with the solid black scaling relationship helps define the span of  $HR$  estimates suggested by the data and assumed mechanism error. In (a), the scaling relationship for the normal distribution with zero mean suggested by Smith and Heaton (2011) is recalculated and plotted with  $HR$  in the 1.08–3.14 range. In (b), a similar scaling relationship is calculated for the mechanism error distribution suggested by the repeating earthquake data with  $HR$  in the 0.92–2.62 range, an approximately 8%–10% decrease. In (c), a scaling relationship with a mechanism error distribution similar to Hardebeck (2015) yields an  $HR$  in the 0.44–2.20 range, an approximately 35%–45% decrease. The Apple Valley region experiences a slightly higher decrease of 60%. The color version of this figure is available only in the electronic edition.



Table 1  
The Effect of Mechanism Error Distributions on the Estimates of Heterogeneity Ratio (*HR*)

	Average Angular Difference	<i>HR</i> for Normal Distribution with Mean of Zero	<i>HR</i> for Distribution from Repeating Earthquakes	Percent Change Decrease from Normal Distribution	<i>HR</i> for Distribution Similar to Hardebeck (2015)	Percent Change Decrease from Normal Distribution
Ventura basin	57.7	1.90	1.74	8.1	1.26	33.7
San Gabriel Mountains	61.2	2.42	2.23	8.0	1.69	30.2
Los Angeles basin	62.1	2.62	2.40	8.3	1.84	29.8
Apple Valley	46.2	1.08	0.96	11.0	0.44	59.8
Landers	63.8	3.14	2.86	9.0	2.20	29.9
Banning	58.7	2.02	1.86	7.9	1.36	32.4
Palm Springs	62.9	2.83	2.59	8.4	1.99	29.6
Coachella	56.9	1.81	1.66	8.1	1.18	34.6
Northern Elsinore	53.3	1.49	1.36	8.4	0.89	40.0
Central Elsinore	53.0	1.47	1.34	8.6	0.87	40.6
Anza	54.0	1.55	1.42	8.7	0.94	39.0
Borrego	54.4	1.58	1.45	8.5	0.98	38.3

that both the data analyzed by Hardebeck (2006) and the power-law filtered stress heterogeneity generated by Smith and Heaton (2011) suggest that stress is more correlated at small interevent distances up to an outer scale. The grid-search inversion of the synthetic focal-mechanism data suggests a spatial smoothing parameter of  $\alpha$  in the 0.8–1.0 range. If  $\alpha = 1.5$ , there would be no biasing at all because the stress would be almost homogeneous. On the other hand, if  $\alpha = 0.0$ , then every point is uncorrelated with the other and biasing would occur uniformly everywhere regardless of the length scale. However, for  $\alpha$  ranging from 0.8 to 1.0, there is heterogeneity but at a small enough length scale that it looks relatively homogeneous. In this case, some regions actively fail and others do not, depending upon their initial stress amplitude and orientation. We therefore hypothesize that shrinking the dimension of the stress inversion region will result in less stress inversion biasing because there is less stress variation within the smaller regions. In other words, as the size of the stress inversion region shrinks to zero, the variation in the stress becomes zero, and the stress inversion biasing disappears within individual regions. Interestingly, as we go toward this limit, we now have a biased sampling of which regions fail where the regions are points, which is exactly the biased sampling of individual events as in the original model of Smith and Heaton (2011).

To test this, we select 2D synthetic focal-mechanism catalogs from Smith and Heaton (2011) and calculate normalized stress inversion biasing for different sizes of subregions. We use the best grid-search result for the repeating earthquake error distribution ( $\alpha = 0.9$ ;  $HR = 2.0$ ). Using catalogs with 10 different random seeds, we randomly subdivide the simulation space for each catalog into smaller subregions. Then we randomly sample the events within each subregion and add random mechanism error consistent with repeating earthquake statistics. This random subdividing and resampling is repeated at least 50 times. Last, we calculate a normalized stress inversion biasing similar to the percent biasing statistic presented in Smith and Heaton (2011), where it represents the percent bias-

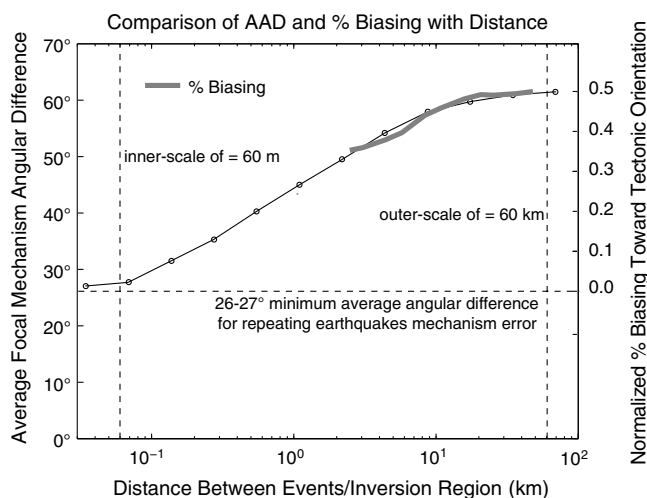
ing toward the stressing rate tensor. In this case, it is normalized because, for a small number of events in the subregions, sometimes the percent biasing is larger due to random mechanism error. Therefore, we use the summation of two statistics from Smith and Heaton (2011), the mean  $\psi^{BI}/\psi^{BT}$  and the mean  $\psi^{IT}/\psi^{BT}$  (the sum of which should equal one in the absence of noise), to normalize our estimates of biasing. Then we plot the results in Figure 4.

As defined in equation (23) of Smith and Heaton (2011), note that  $\psi^{BI}$  is the four component angular difference between two deviatoric stress tensors, the spatial mean background stress tensor and the stress tensor from inversion of focal-mechanism data. The stress inversion is calculated using Michael's program *slick* (see Data and References) based on Michael (1984, 1987).  $\psi^{BT}$  is the angular difference between the background stress tensor and the tectonic loading stress tensor. Last,  $\psi^{IT}$ , is the angular difference between the tectonic stress tensor and the inverted stress tensor.

We find the following: (1) Normalized stress inversion biasing decreases as the size of the subregion decreases. (2) The percent of subregions with sufficient events to fail dramatically decreases; therefore, at the smaller dimensions, only a small fraction of the subregions fail. (3) The fall-off of normalized stress inversion biasing with decreasing spatial scale follows the fall-off of AAD with decreasing interevent distance simulated by Smith and Heaton (2011) and calculated by Hardebeck (2006). At the same time, due to limitations in our simulation, it is difficult to take the region size below 2 km. Regional stress inversion studies may face similar limitations; therefore, our estimates of stress inversion biasing in the previous section should hold for most regional stress inversions.

## Conclusions

We appreciate the work Hardebeck (2015) has done in demonstrating that spherical geometry requires a mechanism error distribution with a nonzero mean. Her suggestion of a more realistic mechanism error distribution has been a



**Figure 4.** Approximation of the Hardebeck (2006) calculation of AAD between pairs of focal mechanisms as a function of interevent distance for southern California A and B quality mechanisms (thin line), and the calculated normalized stress inversion biasing for  $\alpha = 0.9$  and  $HR = 2.0$  and focal-mechanism error consistent with repeating earthquake observations (thick gray line). The modeled normalized stress inversion biasing toward the tectonic stressing rate depends on the outer size of the inversion region. We assume that no biasing occurs when there is little to no heterogeneity at the inner scale and that the maximum biasing occurs at the outer scale. If the outer scale of the stress is 60 km, as suggested by Smith and Heaton (2011), then the stress inversion biasing is about 50% for subregions  $\geq 60$  km and decreases as the inversion subregions decrease. At a subregion size of 3 km, the normalized stress inversion biasing is approximately 35%. However, at the same time, as the size of the subregions decreases, the percentage of the subregions that have sufficient number of events to fail also decreases. Consequently, the seismicity becomes a biased sampling of which subregions fail where these failing subregions represent local stress variations. The figure was modified from Smith and Heaton (2011), which was originally modified from Hardebeck (2006).

helpful contribution. We thank her and Peter Shearer for sharing the Northridge repeating earthquake catalog (Shearer *et al.*, 2003).

We suggest that the focal-mechanism error derived from repeating earthquake data (our upper bound for  $HR$ ) is the best error for angular differences between pairs of events, especially for small interevent distances. Indeed, the relative error between pairs of mechanisms close to one another may be significantly less than the absolute error for individual mechanisms. This relative uncertainty most likely increases as the interevent distance increases; hence, we consider the absolute uncertainty estimated by Hardebeck (2015) for individual mechanisms to be the upper limit on mechanism error and more applicable for the outer-scale limit of the simulations.

When applying a grid search in Figure 2 with our preferred repeating earthquake error distribution,  $HR$  is reduced by only 16% (from 2.375 to 2.0) from the Smith and Heaton (2011) value, and  $\alpha$  is slightly increased from 0.8 to 0.9. The error distribution suggested by Hardebeck (2015) suggests an  $HR$  of approximately 1.5 and an  $\alpha = 1.0$ , but it has trouble fitting the small interevent data well. When using these

new error statistics to estimate  $HR$  for the 12 subregions, we find that  $HR$  is decreased anywhere from 8%–11% (for our preferred statistics from repeating earthquake clusters) to 35%–45% (for the mechanism error distribution suggested by Hardebeck, 2015). We consider these estimates to provide upper and lower bounds on the stress heterogeneity. In our updated estimates, the biasing toward the stressing rate as in Smith and Heaton (2011) can be anywhere from 15% to 50% for the lower  $HR$  bound and anywhere from 30% to 60% for the upper  $HR$  bound, which we prefer.

Although this has helped refine the estimated values, we also find the following major points of Smith and Heaton (2011) to still be valid: (1) focal-mechanism statistics are consistent with stress heterogeneity that is more highly correlated at small length scales and less correlated at longer length scales, and (2) stress inversions applied to regions of 10 km or greater width can exhibit significant biasing toward the stressing rate.

The stress heterogeneity model used by Smith and Heaton (2011) and in this reply is a very simple heterogeneity model. It would be surprising if the real Earth can be represented by so few parameters. In fact, there could easily be time dependence to the heterogeneity amplitude and a spatial dependence with distance from major fault traces. Ultimately, the best way to measure heterogeneous stress is to look at borehole breakout data, which is the actual measurement of stress orientation variability over the length scale of meters. Recent data, such as figure 2 from Day-Lewis (2010), show maximum horizontal stress rotations of  $45^\circ$  over length scales of order 20 m in the San Andreas Fault Observatory at Depth (SAFOD) Pilot Hole, indicative of larger stress variability over short length scales close to the fault. This may show even greater variability of stress than what our model contains; therefore, how much stress variability there is in the real Earth remains an outstanding question to be further refined and studied.

## Data and Resources

The focal-mechanism data used were A and B quality events from the 1984 to 2003 southern California data set (Hardebeck and Shearer, 2003). This catalog is documented at the website [www.data.scec.org/research/altcatalogs.html](http://www.data.scec.org/research/altcatalogs.html) (last accessed May 2014). Repeating earthquake data for the Northridge aftershocks were obtained from Shearer and Hardebeck (Shearer *et al.*, 2003). Some fitting of the hyperbolic functions was aided by Kaleidagraph. The *circm\_vrmd.m* Circular Statistics Toolbox by Philip Berens was obtained from [http://www.mathworks.com/matlabcentral/fileexchange/10676-circular-statistics-toolbox-directional-statistics-content/circ\\_vrmd.m](http://www.mathworks.com/matlabcentral/fileexchange/10676-circular-statistics-toolbox-directional-statistics-content/circ_vrmd.m) (last accessed November 2014). All other code was developed using MATLAB ([www.mathworks.com/products/matlab](http://www.mathworks.com/products/matlab); last accessed November 2014) and run on MacPro computers with OS X. The program *slick* by Michael was obtained from <http://earthquake.usgs.gov/research/software/> (last accessed December 2014).

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We thank Jeanne Hardebeck and Peter Shearer for sharing the Northridge aftershock repeating earthquake cluster catalog with us. We also thank Jeanne Hardebeck and the Southern California Earthquake Center (SCEC) for the focal-mechanism catalog of southern California events.

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